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The Game of Cops and Robbers on Graphs

Anthony Bonato
Richard J. Nowakowski

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Dedicated to the memory of
Paolo Giovanni Bonato and Marian Jozef Nowakowski

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Preface

You are reading a book about a game. More specifically, the game Cops and Robbers, which is played on a graph. Cops and Robbers, in the form we study it, was first introduced in the early 1980s, and a robust body of work on the topic has been growing steadily ever since. At its core, it is a game played with a set of cops (controlled by one player) trying to capture the robber (controlled by the opposing player). The cops and the robber are restricted to vertices, and they move each round to neighboring vertices. The smallest number of cops needed to capture the robber is the *cop number*. Such a simple-sounding game leads to quite a complex theory, as you will learn. A formal introduction to the game and the cop number is given in Chapter 1. Despite the fact that the game is nearly three decades old, the last five years however, have seen an explosive growth in research in the field. Some newer work settles some old problems, while novel approaches, both probabilistic, structural, and algorithmic, have emerged on this classic game on graphs.

We present a book which surveys all of the major developments (both historical and recent) on the topic of Cops and Robbers. As the moniker "Cops and Robbers" represents a class of games with varying rules, we emphasize that we primarily study the game where the cops and robber have perfect information, may only move to neighboring vertices, and move at unit speed (a player can only move at distance

at most one at any step of the game). There is a large and growing literature on variants of the Cops and Robbers game, where there is some notion of "good guys" versus "bad guys". For example, there are versions where there is imperfect information, players can occupy edges or only a subset of vertices, move at faster speeds, or the cops are trying to stop or contain a fire, disease, or contaminant spreading in a graph.

Although these games are not our main focus, we do discuss some variants in Chapters 8 and 9. There are a number of reasons why we wrote this book. One of our goals was to bring together all the most important results, problems, and conjectures in one place to serve as a reference. Hence, this book will be both invaluable to researchers in the field and their students, and a one-stop shop for the major results in the field. We also wanted the book to be self-contained and readable to an advanced undergraduate or beginner graduate student; on the other hand, there are enough advanced topics to either intrigue the seasoned mathematician or theoretical computer scientist. The book is designed to be used either in a course or for independent reading and study. The only prerequisites would be a first course in graph theory, though some mathematical maturity and some background on sets, probability, and algorithms would be helpful. One of our principal goals is to showcase the beauty of the topic, with the ultimate aim of preserving it for the next generations of graph theorists and computer scientists. We also showcase the most challenging open problems in the field. For example, Meyniel's conjecture on upper bounds for the cop number (see Chapter 3) is a deep problem which deserves to be better known.

We now give a summary of the chapters. Chapter 1 supplies all the requisite motivation, notation, basic results, and examples for what comes later. We give a lower bound of Aigner and Fromme on the cop number in terms of girth and minimum degree, and we give the asymptotic upper bound on the cop number supplied by Frankl. Along the way, we discuss guarding isometric paths, and retracts and their critical connections to the game. In Chapter 2 we consider some

new and old characterizations of k-cop-win graphs. We describe in detail the classic characterization of Nowakowski and Winkler and Quilliot of finite cop-win graphs. This beautiful characterization reduces the problem to the existence of a certain ordering of the vertices, a so-called cop-win ordering. It also leads to a strategy for catching the robber called the cop-win strategy. We survey the recent characterization of graphs with cop number $k > 1$ by Clarke and MacGillivray. This characterization uses, among other things, properties of graph products. Chapter 3 is all about Meyniel's conjecture, which concerns an upper bound on the cop number in connected graphs. We give some recent upper bounds and discuss the state-of-the-art on the conjecture. We present a recent proof of the conjecture in the special case of graphs of diameter at most 2. Chapter 4 focuses on the game in graph classes and graph products. We consider bounds for the cop number and related parameters for various products, such as the Cartesian, strong, categorical, and lexicographic products. A proof of the fact that the cop number of planar graphs is at most three is given, and graphs with higher genus are also discussed.

In Chapter 5, we consider algorithmic results on computing the cop number. After an introduction to the rudiments of complexity theory and graph algorithms, we prove that the problem of computing whether the cop number is at most k is in polynomial time, if k is fixed. If k is not fixed, we sketch the proof of the recent result that the problem is **NP**-hard. In Chapter 6 we investigate the cop number in random graphs. We present results for the cop number of the binomial random graph $G(n, p)$, when p is constant, and also consider recent work in case $p = p(n)$ is a function of n. We culminate with the beautiful Zig-Zag Theorem of Łuczak and Prałat, which reveals a surprising, literal twist to the behaviour of the cop number in random graphs. We finish with a study of the cop number in models for the web graph and other complex networks. In Chapter 7 we study the game of Cops and Robbers played in infinite graphs. Infinite graphs often exhibit unusual properties not seen in the finite case; the cop number in the infinite case is no exception to this. We introduce the cop density of a countable graph and show that the cop density of the infinite random graph can be any real number in $[0, 1]$. We survey

the results of Hahn, Sauer, and Woodrow on infinite chordal cop-win graphs. We finish the chapter with a discussion of paradoxically large families of infinite vertex-transitive cop-win graphs. Chapters 8 and 9 consider variants of the game, and are more like surveys when compared with previous chapters. In Chapter 8, we consider the effect of changing the rules of Cops and Robbers. In particular, we consider imperfect information where the robber is partially invisible, and the inclusion of traps, alarms, and photo radar. We consider tandem-cops where cops must always be sufficiently close to each other during the course of the game. In addition, we consider a version of Cops and Robbers where the cops can capture the robber from some prescribed distance (akin to shooting the robber), and we investigate the length of time it takes for the cops to win assuming optimal play. At the heart of all the games we consider, there is the notion of a set of *good guys* trying to stop, contain, or capture a *bad guy*. Chapter 9 deals with several of these kinds of games, including firefighting, edge searching, Helicopter Cops and Robbers, graph cleaning, and robot vacuum. We conclude with a brief section on combinatorial games.

We think the book would make a solid second or topics course in graph theory. An ambitious (likely two-term) course would cover all nine chapters. For a one-term course, we suggest three options: generalist and specialist courses, along with an experiential option. A *generalist* course would cover each of the first four chapters and two of the remaining ones. Such a course would give a solid grounding in the field and supply some flexibility at the end, depending on the tastes of the instructor and audience. A *specialist* course would cover the first two chapters, one of Chapters 3 or 4, and three of the last five chapters. This option would appeal to those would like to learn one of the more advanced topics (such as algorithms, random or infinite graphs) in greater detail. Finally, an *experiential* course would cover Chapters 1, 2, 4, 8, and 9. The emphasis in such a course would be on projects, filling in omitted proofs (Chapters 4, 8, and 9 contain surveys with proofs omitted), coming up with new examples, and developing new variants of Cops and Robbers. This last option would be especially useful in the setting of a summer research project (such as one sponsored by an NSERC USRA or NSF REU).

To both aid and challenge the reader, there are over 200 exercises in the book, with many worked examples throughout. Open problems are cited in the exercises and elsewhere. We will maintain a website

`http://www.math.ryerson.ca/~abonato/copsandrobbers.html`

which will contain resources such as errata and lists of open problems. Hopefully, it will also contain their eventual solutions!

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Bibliography

[1] I. Adler, Marshals, monotone Marshals, and hypertree-width, *Journal of Graph Theory* **47** (2004) 275–296.

[2] M. Aigner, M. Fromme, A game of cops and robbers, *Discrete Applied Mathematics* **8** (1984) 1–11.

[3] M.H. Albert, R.J. Nowakowski, D. Wolfe, *Lessons in Play,* A K Peters, Ltd., 2007.

[4] R. Albert, H. Jeong, A. Barabási, Diameter of the world-wide web, *Nature* **401** (1999) 130.

[5] N. Alon, P. Prałat, N. Wormald, Cleaning regular graphs with brushes, *SIAM Journal on Discrete Mathematics* **23** (2008/09) 233–250.

[6] N. Alon, J. Spencer, *The Probabilistic Method*, Wiley, New York, 2000.

[7] B. Alspach, Sweeping and searching in graphs: a brief survey, *Matematiche* **59** (2006) 5–37.

[8] B. Alspach, D. Dyer, D. Hanson, B. Yang, Lower bounds on edge searching, *ESCAPE 2007*, B. Chen, M. Paterson, G. Zhang (Eds.), **4614** (2007) 516–527.

[9] B. Alspach, X. Li, B. Yang, Searching graphs and directed graphs, preprint.

[10] T. Andreae, Note on a pursuit game played on graphs, *Discrete Applied Mathematics* **9** (1984) 111–115.

[11] T. Andreae, On a pursuit game played on graphs for which a minor is excluded, *Journal of Combinatorial Theory, Series B* **41** (1986) 37–47.

[12] R.P. Anstee, M. Farber, On bridged graphs and cop-win graphs, *Journal of Combinatorial Theory, Series B* **44** (1988) 22–28.

[13] K. Appel, W. Haken, J. Koch, Every planar map is four colorable, *Illinois Journal of Mathematics* **21** (1977) 439-567.

[14] W. Baird, *Cops, Robbers, and Graphs*, M.Sc. Thesis, Ryerson University, 2011.

[15] W. Baird, A. Bonato, Meyniel's conjecture on the cop number: a survey, Preprint 2011.

[16] A. Berarducci, B. Intrigila, On the cop number of a graph, *Advances in Applied Mathematics* **14** (1993) 389–403.

[17] E.R. Berlekamp, J.H. Conway, R.K. Guy, *Winning ways for your mathematical plays,* Volumes 1, 2, 3, and 4, Second Editions, A K Peters, Ltd., 2001–2004.

[18] B. Bollobás, *Random graphs, Second edition,* Cambridge Studies in Advanced Mathematics, **73**, Cambridge University Press, Cambridge, 2001.

[19] B. Bollobás, G. Kun, I. Leader, Cops and robbers in a random graph, Preprint 2011.

[20] B. Bollobás, I. Leader, The Angel and the Devil in three dimensions, *Journal of Combinatorial Theory, Series A* **113** (2006) 176–184.

[21] A. Bonato, *A Course on the Web Graph*, Graduate Studies in Mathematics, American Mathematical Society, Providence, Rhode Island, 2008.

[22] A. Bonato, E. Chiniforooshan, Pursuit and evasion from a distance: algorithms and bounds, In: *Proceedings of Workshop on Analytic Algorithms and Combinatorics (ANALCO'09)*, 2009.

[23] A. Bonato, E. Chiniforooshan, P. Prałat, Cops and Robbers from a distance, *Theoretical Computer Science* **411** (2010) 3834–3844.

[24] A. Bonato, D. Delić, On a problem of Cameron's on inexhaustible graphs, *Combinatorica* **24** (2004) 35–51.

[25] A. Bonato, N. Hadi, P. Horn, P. Prałat, C. Wang, Models of on-line social networks, *Internet Mathematics* **6** (2011) 285–313.

[26] A. Bonato, G. Hahn, P.A. Golovach, J. Kratochvíl, The capture time of a graph, *Discrete Mathematics* **309** (2009) 5588–5595.

[27] A. Bonato, G. Hahn, C. Tardif, Large classes of infinite k-cop-win graphs, *Journal of Graph Theory* **65** (2010) 334–242.

[28] A. Bonato, G. Hahn, C. Wang, The cop density of a graph, *Contributions to Discrete Mathematics* **2** (2007) 133–144.

[29] A. Bonato, G. Kemkes, P. Prałat, Almost all cop-win graphs contain a universal vertex, Preprint 2011.

[30] A. Bonato, M.E. Messinger, P. Prałat, Fighting intelligent fires in graphs, Preprint 2011.

[31] A. Bonato, P. Prałat, C. Wang, Pursuit-evasion in models of complex networks, *Internet Mathematics* **4** (2009) 419–436.

[32] B.H. Bowditch, The Angel Game in the plane, *Combinatorics, Probability and Computing,* **16** (2007) 345–362.

[33] A. Brandstädt, V.B. Le, J.P. Spinrad, *Graph Classes: A Survey*, SIAM Monographs on Discrete Mathematics and Applications, Philadelphia, 1999.

[34] G. Brightwell, P. Winkler, Gibbs measures and dismantlable graphs, *Journal of Combinatorial Theory, Series B* **78** (2000) 141–169.

[35] A. Broder, R. Kumar, F. Maghoul, P. Raghavan, S. Rajagopalan, R. Stata, A. Tomkins, J. Wiener, Graph structure in the web, *Computer Networks* **33** (2000) 309–320.

[36] L. Cai, W. Wang, The safety factor of square grids, preprint.

[37] L. Cai, W. Wang, The surviving rate of a graph, *SIAM Journal of Discrete Mathematics*, **23** (2009) 1814–1826.

[38] L. Cai, Y. Cheng, E. Verbin, Y. Zhou, Surviving rates of trees and outerplanar graphs for the Firefighter problem, *SIAM Journal on Discrete Mathematics* **24** (2010) 1322–1335.

[39] P.J. Cameron, *Combinatorics: Topics, Techniques, Algorithms*, Cambridge University Press, Cambridge, 1995.

[40] P.J. Cameron, The random graph, In: *The Mathematics of Paul Erdős*, *II*, Algorithms and Combinatorics, **14**, Springer, Berlin, 1997, pp. 333–351.

[41] G. Campbell, On optimal play in the game of Hex, *INTEGERS* **4** (2004) #G2, 23pp.

[42] G. Cantor, Beiträge zur begründung der transfiniten mengenlehre, *Mathematische Annalen* **49** (1897) 207–246.

[43] J. Chalopin, V. Chepoi, N. Nisse, Y. Vaxés, Cop and robber games when the robber can hide and ride, Technical Report, *INRIA-RR7178*, Sophia Antipolis, France, Jan. 2010.

[44] M. Chastand, F. Laviolette, N. Polat, On constructible graphs, infinite bridged graphs and weakly cop-win graphs, *Discrete Mathematics* **224** (2000) 61–78.

[45] P. Chebyshev, Mémoire sur les nombres premiers, *Mém. Acad. Sci. St. Pétersbourg* **7** (1850) 17–33.

[46] V. Chepoi, Bridged graphs are cop-win graphs: an algorithmic proof, *Journal of Combinatorial Theory, Series B* **69** (1997) 97–100.

[47] E. Chiniforooshan, A better bound for the cop number of general graphs, *Journal of Graph Theory* **58** (2008) 45–48.

[48] M. Chudnovsky, N. Robertson, P. Seymour, R. Thomas, The strong perfect graph theorem, *Annals of Mathematics* **164** (2006) 51–229.

[49] F.R.K. Chung, L. Lu, *Complex graphs and networks*, American Mathematical Society, Providence RI, 2006.

[50] N.E. Clarke, A witness version of the Cops and Robber game, accepted to *Discrete Mathematics*.

[51] N.E. Clarke, *The effects of replacing cops and searchers with technology*, M.Sc. Thesis, Dalhousie University, 1999.

[52] N.E. Clarke, *Constrained Cops and Robber*, Ph.D. Thesis, Dalhousie University, 2002.

[53] N.E. Clarke, A game of Cops and Robber played with partial information, *Congressus Numerantium* **166** (2004) 145–159.

[54] N.E. Clarke, E.L. Connon, Cops, Robber, and alarms, *Ars Combinatoria* **81** (2006) 283–296.

[55] N.E. Clarke, S. Finbow, S.L. Fitzpatrick, M.E. Messinger, R.J. Nowakowski, Seepage in directed acyclic graphs, *Australasian Journal of Combinatorics* **43** (2009) 91–102.

[56] N.E. Clarke, S.L. Fitzpatrick, A. Hill, R.J. Nowakowski, Edge critical Cops and Robber, accepted to *Discrete Mathematics*.

[57] N.E. Clarke, G. MacGillivray, Characterizations of k-copwin graphs, Preprint 2011.

[58] N.E. Clarke, R.J. Nowakowski, Cops, Robber and photo radar, *Ars Combinatoria* **56** (2000), 97–103.

[59] N.E. Clarke, R.J. Nowakowski, Cops, robber, and traps, *Utilitas Mathematica* **60** (2001) 91–98.

[60] N.E. Clarke, R.J. Nowakowski, A tandem version of the Cops and Robber game played on products of graphs, *Discussiones Mathematicae Graph Theory* **25** (2005) 241–249.

[61] N.E. Clarke, R.J. Nowakowski, Tandem-win graphs, *Discrete Mathematics* **299** (2005) 56–64.

[62] Clay Mathematics Institute Millenium Prize Problems. Accessed March 31, 2011. `http://www.claymath.org/millennium/`

[63] J.H. Conway, The angel problem, *Games of No Chance*, R.J. Nowakowski editor, Cambridge University Press, 1996, pp. 3–12.

[64] S. Cook, The complexity of theorem proving procedures, In: *Proceedings of the Third Annual ACM Symposium on Theory of Computing*, 1971.

[65] R. Crandall, C. Pomerance, *Prime Numbers: A Computational Perspective, 2nd ed.* Springer-Verlag, New York, 2005.

[66] R.M. Damerell, On Moore graphs, *Proceedings of the Cambridge Philosophical Society* **74** (1973) 227–236.

[67] M. Devlin, S. Hartke, Fire containment in grids of dimension three and higher, *Discrete Applied Mathematics* **155** (2007) 2257–2268.

[68] R. Diestel, *Graph theory*, Springer-Verlag, New York, 2000.

[69] R.P. Dilworth, A decomposition theorem for partially ordered sets, *Annals of Mathematics* **51** (1950) 161–166.

[70] P.A. Dreyer, Applications and variations of domination in graphs, Ph.D. Dissertation, Department of Mathematics, Rutgers University, 2000.

[71] J.A. Ellis, I.H. Sudborough, J.S. Turner, The vertex separation and search number of a graph, *Information and Computation* **113** (1994) 50–79.

[72] P. Erdős, Beweis eines Satzes von Tschebyschef, *Acta Sci. Math. (Szeged)* **5** (1930-32) 194–198.

[73] P. Erdős, A. Rényi, On random graphs I, *Publicationes Mathematicae Debrecen* **6** (1959) 290–297.

[74] P. Erdős, A. Rényi, On the evolution of random graphs, *Publ. Math. Inst. Hungar. Acad. Sci.* **5** (1960) 17–61.

[75] P. Erdős, A. Rényi, Asymmetric graphs, *Acta Mathematica Academiae Scientiarum Hungaricae* **14** (1963) 295–315.

[76] S. Finbow, G. MacGillivray, The Firefighter problem: a survey of results, directions and questions, *Australasian Journal of Combinatorics* **43** (2009) 57–77.

[77] S. Finbow, A.D. King, G. MacGillivray, R. Rizzi, The Firefighter problem for graphs of maximum degree three, *Discrete Mathematics* **307** (2007) 2094–2105.

[78] S. Finbow, P. Wang, W. Wang, On the surviving rate of a graph, preprint.

[79] S.L. Fitzpatrick, *Aspects of domination and dynamic domination*, Ph.D. Thesis, Dalhousie University, 1997.

[80] S.L. Fitzpatrick, R.J. Nowakowski, Copnumber of graphs with strong isometric dimension two, *Ars Combinatoria* **59** (2001) 65–73.

[81] S.L. Fitzpatrick, R.J. Nowakowski, D. Holton, I. Caines, Covering hypercubes by isometric paths, *Discrete Mathematics* **240** (2001) 253–260.

[82] P. Fogarty, *Catching the fire on grids*, M.Sc. Thesis (2003), Department of Mathematics, University of Vermont.

[83] S. Foldes, P.L. Hammer, Split graphs, In: *Proceedings of the 8th South-Eastern Conference on Combinatorics, Graph Theory and Computing*, 1977.

[84] F.V. Fomin, P. Goldvach, J. Kratochvíl, On tractability of the Cops and Robbers game. In: *Fifth IFIP International Conference On Theoretical Computer Science- TCS 2008, IFIP 20th World Computer Congress, TC 1, Foundations of Computer Science*, 2008.

[85] F.V. Fomin, P.A. Golovach, J. Kratochvíl, N. Nisse, Pursuing fast robber in graphs, *Theoretical Computer Science* **411** (2010) 1167–1181.

[86] F.V. Fomin, P.A. Golovach, D. Lokshtanov, Cops and Robber game without recharging, In: *Proceedings of 12th Scandinavian Symposium and Workshops on Algorithm Theory (SWAT 2010)*, 2010.

[87] F.V. Fomin, D.M. Thilikos, An annotated bibliography on guaranteed graph searching, *Theoretical Computer Science* **399** (2008) 236–245.

[88] P. Frankl, On a pursuit game on Cayley graphs, *Combinatorica* **7** (1987) 67–70.

[89] P. Frankl, Cops and robbers in graphs with large girth and Cayley graphs, *Discrete Applied Mathematics* **17** (1987) 301–305.

[90] A. Frieze, M. Krivelevich, P. Loh, Variations on Cops and Robbers, accepted to *Journal of Graph Theory*.

[91] A. Gajardo, A. Moreira, E. Goles, Complexity of Langton's ant, *Discrete Applied Mathematics* **117** (2002) 41–50.

[92] A. Gardiner, Homogeneous graphs, *Journal of Combinatorial Theory, Series B* **20** (1976) 94–102.

[93] M. Gardner, *Hexaflexagons and Other Mathematical Diversions*, University of Chicago Press, Chicago, 1988.

[94] M. Garey, D. Johnson, *Computers and Intractability, A Guide to the Theory of NP-Completeness*, Freemann, San Francisco, 1979.

[95] S. Gaspers, M.E. Messinger, R.J. Nowakowski, P. Prałat, Parallel cleaning of a network with brushes, *Discrete Applied Mathematics* **158** (2010) 467–478.

[96] S. Gaspers, M.E. Messinger, R.J. Nowakowski, P. Prałat, Clean the graph before you draw it! *Information Processing Letters* **109** (2009) 463–467.

[97] T. Gavenčiak, *Games on graphs*, Master's thesis, Department of Applied Mathematics, Charles University, Prague, 2007.

[98] T. Gavenčiak, Cop-win graphs with maximal capture-time, *Studentská vědecká a odbornáčinnost (SVOC)*, 2008.

[99] Y. Gol'fand, M. Klin, On k-homogeneous graphs, *Algorithmic Studies in Combinatorics*, Work Collect., Moskva (1978) 76–85.

[100] A.S. Goldstein, E.M. Reingold, The complexity of pursuit on a graph, *Theoretical Computer Science* **143** (1995) 93–112.

[101] P. Gordinowicz, P. Prałat, personal communication.

[102] G.R. Grimmett, D.R. Stirzaker, *Probability and Random Processes*, 3rd Edition, Oxford University Press, 2001.

[103] G. Hahn, Cops, robbers and graphs, *Tatra Mountain Mathematical Publications* **36** (2007) 163–176.

[104] G. Hahn, F. Laviolette, N. Sauer, R.E. Woodrow, On cop-win graphs, *Discrete Mathematics* **258** (2002) 27–41.

[105] G. Hahn, G. MacGillivray, A characterization of k-cop-win graphs and digraphs, *Discrete Mathematics* **306** (2006) 2492–2497.

[106] Y.O. Hamidoune, On a pursuit game on Cayley digraphs, *European Journal of Combinatorics* **8** (1987) 289–295.

[107] F. Harary, G.W. Wilcox, Boolean operations on graphs, *Mathematica Scandinavica* **20** (1967) 41–51.

[108] B. Hartnell, Firefighter! An application of domination. Presentation at the *25th Manitoba Conference on Combinatorial Mathematics and Computing*, University of Manitoba, Winnipeg, Canada, 1995.

[109] B. Hartnell, Q. Li, Firefighting on trees: How bad is the greedy algorithm?, In: *Proceedings of the Thirty-first Southeastern International Conference on Combinatorics, Graph Theory and Computing*, 2000.

[110] B. Hartnell, D. Rall, On Vizing's Conjecture, *Congressus Numerantium* **82** (1991) 87–96.

[111] P. Hell, I. Rival, Absolute retracts and varieties of reflexive graphs, *Canadian Journal of Mathematics* **39** (1987) 544–567.

[112] C.W. Henson, A family of countable homogeneous graphs, *Pacific Journal of Mathematics* **38** (1971) 69–83.

[113] A. Hill, *Cops and Robbers: Theme and Variations*, Ph.D. Thesis, Dalhousie University, 2008.

[114] B. Hobbs, J. Kahabka, Underwater Cleaning Technique Used for Removal of Zebra Mussels at the Fitzpatrick Nuclear Power Plant, In: *Proceedings of The Fifth International Zebra Mussel and Other Aquatic Nuisance Organisms Conference, The Sea Grant Nonindigenous Species Site*. 1995.

[115] A.J. Hoffman, R.R. Singleton, On Moore graphs with diameter 2 and 3, *IBM Journal of Research and Development* **4** (1960) 497–504.

[116] J.E. Hopcroft, R. Motwani, J.D. Ullman, *Introduction to Automata Theory, Languages, and Computation, 3rd Edition*, Addison-Wesley, 2006.

[117] P. Hunter, S. Kreutzer, Digraph measures: Kelly decompositions, games, and orderings, *Theoretical Computer Science* **399** (2008) 206–219.

[118] W. Imrich, H. Izbicki, Associative Products of Graphs, *Monatshefte fur Mathematik* **80** (1975) 277–281.

[119] W. Imrich, S. Klavzar, *Product graphs, Structure and Recognition*, Wiley-Interscience Series in Discrete Mathematics and Optimization. Wiley-Interscience, New York, 2000.

[120] A. Isaza, J. Lu, V. Bulitko, R. Greiner, A cover-based approach to multi-agent moving target pursuit, In: *Proceedings of The 4th Conference on Artificial Intelligence and Interactive Digital Entertainment*, 2008.

[121] V. Isler, S. Kannan, S. Khanna, Randomized Pursuit-Evasion with Local Visibility, *SIAM Journal on Discrete Mathematics* **1** (2006) 26–41.

[122] V. Isler, S. Kannan, S. Khanna, Randomized Pursuit-Evasion in a Polygonal Environment, *IEEE Transactions on Robotics* **21** (2005) 864–875.

[123] S. Janson, T. Łuczak, A. Ruciński, *Random Graphs*, Wiley, New York, 2000.

[124] D.V. Jeliazkova, *Aspects of the Cops and Robber Game played with incomplete information*, M.Sc. Thesis, Acadia University, 2006.

[125] G. Joret, M. Kamiński, D.O. Theis, The cops and robber game on graphs with forbidden (induced) subgraphs, *Contributions to Discrete Mathematics* **5** (2010) 40–51.

[126] R.M. Karp, Reducibility among combinatorial problems, In: *Complexity of Computer Computations*, Raymond E. Miller and James W. Thatcher (editors), New York: Plenum, pp. 85–103, 1972.

[127] J. Kára, K. Kratochvíl, D. Wood, On the complexity of the balanced vertex ordering problem, *Discrete Mathematics and Theoretical Computer Science* **9** (2007) 193–202.

[128] S. R. Kotler, E.C. Mallen, K.M. Tammus, Robotic Removal of Zebra Mussel Accumulations in a Nuclear Power Plant Screenhouse, *Proceedings of the Fifth International Zebra Mussel and Other Aquatic Nuisance Organisms Conference, The Sea Grant Nonindigenous Species Site*, 1995.

[129] R. Kumar, P. Raghavan, S. Rajagopalan, A. Tomkins, Trawling the web for emerging cyber-communities, In: *Proceedings of the 8th WWW Conference*, 1999.

[130] K. Kuratowski, Sur le problème des courbes gauches en topologie, *Fundamenta Mathematicae* **15** (1930) 271–283.

[131] M. Kutz, Conway's angel in three dimensions, *Theoretical Computer Science* **349** (2005) 443–451.

[132] A.H. Lachlan, R.E. Woodrow, Countable ultrahomogeneous undirected graphs, *Transaction of the American Mathematical Society* **262** (1980) 51–94.

[133] C.W.H. Lam, The search for a finite projective plane of order 10, *American Mathematical Monthly* **98** (1991) 305–318.

[134] C.G. Langton, Studying artificial life with cellular automata, *Physica D: Nonlinear Phenomena* **22** (1986) 120–149.

[135] A.S. LaPaugh, Recontamination does not help to search a graph, *Journal of the Association for Computing Machinery* **40** (1993) 224–245.

[136] A. Lehman, A solution to the Shannon switching game, *SIAM Journal on Applied Mathemtics* **12** (1964) 687–725.

[137] L. Levin, Universal search problems, *Problems of Information Transmission* **9** (1973) 265–266. (Russian)

[138] H. Lewis, C.H. Papadimitriou, *Elements of the Theory of Computation, 2nd Edition*, Prentice Hall, Upper Saddle River, NJ, 1997.

[139] Z. Li, A. Vetta, Bounds on the cleaning times of robot vacuums, *Operations Research Letters* **38** (2010) 69–71.

[140] L. Lovász, Operations with structures, *Acta Mathematica Hungarica* **18** (1967) 321–328.

[141] L. Lu, X. Peng, On Meyniel's conjecture of the cop number, Preprint 2011.

[142] T. Łuczak, P. Prałat, Chasing robbers on random graphs: Zig-Zag theorem, *Random Structures and Algorithms* **37** (2010) 516–524.

[143] E.M. Luks, L. Babai, Canonical labeling of graphs, In: *Proceedings of the 15th ACM Symposium on Theory of Computing*, 1983.

[144] M. Maamoun, H. Meyniel, On a game of policemen and robber, *Discrete Applied Mathematics* **17** (1987) 307–309.

[145] G. MacGillivray, P. Wang, On the Firefighter problem, *Journal of Combinatorial Mathematics and Combinatorial Computing* **47** (2003) 83-96.

[146] A. Mathe, The Angel of power 2 wins, *Combinatorics, Probability and Computing* **16** (2007) 363–374.

[147] N. Megiddo, S.L. Hakimi, M.R. Garey, D.S. Johnson, C.H. Papadimitriou, The complexity of searching a graph, *Journal of the Association for Computing Machinery* **35** (1988) 18–44.

[148] M.E. Messinger, *Firefighting on Infinite Grids*, M.Sc. Thesis, Department of Mathematics and Statistics, Dalhousie University, 2004.

[149] M.E. Messinger, Firefighting on the triangular grid, *Journal of Combinatorial Mathematics and Combinatorial Computing* **63** (2007) 37–45.

[150] M.E. Messinger, Firefighting on the strong grid, Preprint 2011.

[151] M.E. Messinger, *Methods of Decontaminating a Network*, PhD. Thesis, Department of Mathematics and Statistics, Dalhousie University, 2008.

[152] M.E. Messinger, R.J. Nowakowski, The robot cleans up, *Journal of Combinatorial Optimization* **18** (2009) 350–361.

[153] M.E. Messinger, R.J. Nowakowski, P. Prałat, Cleaning a network with brushes, *Theoretical Computer Science* **399** (2008) 191–205.

[154] M.E. Messinger, R.J. Nowakowski, P. Prałat, N. Wormald, Cleaning random regular graphs with brushes using a degree-greedy algorithm, In: *Proceedings of the 4th Workshop on Combinatorial and Algorithmic Aspects of Networking (CAAN 2007)*, 2007.

[155] M.E. Messinger, R.J. Nowakowski, P. Prałat, Cleaning with brooms, *Graphs and Combinatorics* **27** (2011) 251–267.

[156] M. Miller, J. Sirán, Moore graphs and beyond: A survey of the degree/diameter problem, *Electronic Journal of Combinatorics* DS14, 2005.

[157] S.A. Moeller, P. Wang, Fire control on graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing* **41** (2002) 19–34.

[158] C. Moldenhauer, Game tree search algorithms for the game of Cops and Robber, M.Sc. Thesis, University of Edmonton, 2009.

[159] C. Moldenhauer, N. Sturtevant, Evaluating strategies for running from the cops, In: *Proceedings of IJCAI*, 2009.

[160] J.W. Moon, *Topics on Tournaments*, Holt, Rinehart and Winston, New York, 1968.

[161] M. Musson, A. Tang, *Cops and Robber with road blocks*, Preprint 2011.

[162] J. Nešetřil, V. Rödl, Products of graphs and their applications, *Graph Theory, Łagów 1981, Lecture Notes in Mathematics 1018*, Springer, Berlin, pp 151–160, 1983.

[163] S. Neufeld, *The game of Cops and Robber*, M.Sc. Thesis, Dalhousie University, 1990.

[164] S. Neufeld, R.J. Nowakowski, A game of cops and robbers played on products of graphs, *Discrete Mathematics* **186** (1998) 253–268.

[165] S. Neufeld, R.J. Nowakowski, A vertex-to-vertex pursuit game played with disjoint sets of edges, *Finite and infinite combinatorics in sets and logic* (Banff, AB, 1991), Kluwer Acad. Publ., 1993, 411, 299–312

[166] R.J. Nowakowski, D.F. Rall, Associative graph products and their independence, domination and coloring numbers, *Discussiones Mathematicae Graph Theory* **16** (1996) 53–79.

[167] R.J. Nowakowski, P. Winkler, Vertex-to-vertex pursuit in a graph, *Discrete Mathematics* **43** (1983) 235–239.

[168] P. Prałat, Cleaning random graphs with brushes, *Australasian Journal of Combinatorics* **43** (2009) 237–251.

[169] A. Quilliot, Jeux et pointes fixes sur les graphes, *Thèse de 3ème cycle*, Université de Paris VI, 1978, 131–145.

[170] A. Quilliot, *Problèmes de jeux, de point Fixe, de connectivité et de represésentation sur des graphes, des ensembles ordonnés et des hypergraphes*, Thèse d'Etat, Université de Paris VI, 1983, 131–145.

[171] A. Quilliot, A short note about pursuit games played on a graph with a given genus, *Journal of Combinatorial Theory, Series B* **38** (1985) 89–92.

[172] C.H. Papadimitriou, *Computational Complexity*, Addison-Wesley, 1994.

[173] T.D. Parsons, Pursuit-evasion in a graph, *Theory and applications of graphs* (Proc. Internat. Conf., Western Mich. Univ., Kalamazoo, Mich., 1976), Springer, 1978, 426-441. Lecture Notes in Mathematics, Vol. 642.

[174] T.D. Parsons, The search number of a connected graph, Proceedings of the Ninth Southeastern Conference on Combinatorics, Graph Theory, and Computing, (Florida Atlantic Univ., Boca Raton, Fla., 1978), *Utilitas Mathematics* (1978) 549–554.

[175] P. Prałat, When does a random graph have constant cop number?, *Australasian Journal of Combinatorics* **46** (2010) 285–296.

[176] P. Prałat, Cleaning random d-regular graphs with brooms, accepted to *Graphs and Combinatorics*.

[177] P. Prałat, Cleaning random graphs with brushes, *Australasian Journal of Combinatorics* **43** (2009) 237–251.

[178] P. Prałat, Graphs with average degree smaller than 30/11 are burning slowly, Preprint 2011.

[179] P. Prałat, N. Wormald, Meyniel's conjecture holds in random graphs, Preprint 2011.

[180] R. Rado, Universal graphs and universal functions, *Acta Arithmetica* **9** (1964) 331–340.

[181] T. Ramanampanoharana, *Jeu de poursuite sur des modèles du web et généralisations*, Masters thesis, Université de Montréal, 2004.

[182] C.S. ReVelle, K.E. Rosing, Defendens imperium romanum: a classical problem in military strategy, *American Mathematical Monthly* **107** (2000) 585–594.

[183] N. Robertson, D.P. Sanders, P. Seymour, R. Thomas, The four-color theorem, *Journal of Combinatorial Theory, Series B* **70** (1997) 2–44.

[184] E.R. Scheinerman, D.H. Ullman, *Fractional graph theory*, Wiley-Interscience, New York, 1997.

[185] B.S.W. Schroeder, The copnumber of a graph is bounded by $\lfloor \frac{3}{2}\text{genus}(G) \rfloor + 3$, *Categorical Perspectives*, Trends Math., Birkhäuser, Boston, MA, 2001, 243–263.

[186] A. Scott, *On the parameterized complexity of finding short winning strategies in combinatorial games*, Ph.D. Thesis, University of Victoria, 2010.

[187] A. Scott, B. Sudakov, A new bound for the cops and robbers problem, Preprint 2011.

[188] P.D Seymour, R. Thomas, Graph searching and a min-max theorem for tree-width, *Journal of Combinatorial Theory Series B* **58** (1993) 22–33.

[189] J. Sheehan, Smoothly embeddable subgraphs, *Journal of the London Mathematical Society* **9** (1974) 212–218.

[190] A. Tang, *Zero-visibility Cops and Robber*, M.Sc. Thesis, Dalhousie University, 2004.

[191] D.O. Theis, The cops and robber game on series-parallel graphs, accepted to *Graphs and Combinatorics*.

[192] R. Tošíc, On cops and robber game, *Studia Scientiarum Mathematicarum Hungarica* **23** (1988) 225–229.

[193] A. Turing, On computable numbers, with an application to the Entscheidungsproblem, *Proceedings of the London Mathematical Society* **42** (1936) 230–265.

[194] V.V. Vazirani *Approximation Algorithms*, Springer-Verlag, Berlin, 2001.

[195] V.G. Vizing, The cartesian product of graphs, *Vyčisl. Sistemy* **9** (1963) 30–43.

[196] B. Wieland, A.P. Godbole, On the domination number of a random graph, *The Electronic Journal of Combinatorics* **8** (2001), #R37.

[197] D.B. West, *Introduction to Graph Theory, 2nd edition*, Prentice Hall, 2001.

[198] A. Wigderson, P, NP and Mathematics—a computational complexity perspective, In: *Proceedings of the ICM'06*, 2007.

[199] N.C. Wormald, Models of random regular graphs, *Surveys in Combinatorics*, J.D. Lamb and D.A. Preece, eds. London Mathematical Society Lecture Note Series, vol. 276, pp. 239–298, Cambridge University Press, Cambridge, 1999.

[200] N.C. Wormald, The differential equation method for random graph processes and greedy algorithms, *Lectures on Approximation and Randomized Algorithms*, PWN, 1999, 73–155.

[201] B. Yandell, *The Honors Class: Hilbert's Problems and Their Solvers,* A. K. Peters Ltd, Nantick, MA, 2003.

[202] V. Yanovski, I.A. Wagner, A.M. Bruckstein, A distributed ant algorithm for efficiently patrolling a network, *Algorithmica* **37** (2003) 165–186.

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This book is the first and only one of its kind on the topic of Cops and Robbers games, and more generally, on the field of vertex pursuit games on graphs. The book is written in a lively and highly readable fashion, which should appeal to both senior undergraduates and experts in the field (and everyone in between). One of the main goals of the book is to bring together the key results in the field; as such, it presents structural, probabilistic, and algorithmic results on Cops and Robbers games. Several recent and new results are discussed, along with a comprehensive set of references. The book is suitable for self-study or as a textbook, owing in part to the over 200 exercises. The reader will gain insight into all the main directions of research in the field and will be exposed to a number of open problems.

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